A novel fuzzy metaheuristic approach in nurse rerostering problem

SVETLANA SIMIĆ*, *Faculty of Medicine, University of Novi Sad, Hajduk Veljkova 1-9, Novi Sad 21000, Serbia.*

EMILIO CORCHADO**, *Departamento de Informática y Automática, Universidad de Salamanca, Plaza de la Merced, s/n, Salamanca 37008, Spain.*

DRAGAN SIMIĆ†, *Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, Novi Sad 21000, Serbia.*

JOVANKA ĐORĐEVIĆ††, *Oncology Institute of Vojvodina, Put doktora Goldmana 4, 21208 Sremska Kamenica, Serbia.*

SVETISLAV D. SIMIĆ§, *Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, Novi Sad 21000, Serbia.*

Abstract

In the past 50 years nurse scheduling has received considerable attention in the research community. There are two cases regarding the nurse scheduling problem (NSP): the *static* and *dynamic*. Dynamic NSP, is often called *nurse rerostering problem* (NRRP), which presents reconstruction or modification of the predetermined roster for the current scheduling horizon. The aim of this paper is to present new hybrid strategy for nurse NRRP problem. The proposed methodology is based on efficient cooperation between fuzzy logic, ordered weighted averaging and variable neighbourhood descent search. Complete system is tested, and experimental results are based on real-world dataset obtained from the Oncology Institute of Vojvodina in Serbia.

Keywords: nurse rerostering problem, fuzzy logic, ordered weighted averaging model, variable neighbourhood descent search

1. Introduction

Healthcare systems provide round-the-clock medical and paramedical services to the society. Human resources are the key resource in the healthcare organization. Medical staff performance and staff scheduling represent a significant determinant of public healthcare quality where the hospitals are required to exercise cost effectiveness. Oftentimes, these systems operate under dynamic and fuzzy environments in which unanticipated events may take place, leading to disruptions of planned

*E-mail: svetlana.simic@mf.uns.ac.rs
**E-mail: escorchado@usal.es
†E-mail: dsimic@eunet.rs, dsimic@uns.ac.rs
††E-mail: jovankad10@gmail.com
§E-mail: simicsvetislav@uns.ac.rs

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activities. This is commonplace in hospital wards where limited nurses are scheduled 24 hours a day 7 days a week.

There is an intense pressure for cost reduction, which negatively influences work–life balance for a small number of employed physicians and nurses and often results in a decrease of demanded service quality, since they often must take consecutive shifts or cannot take a *day off*. Moreover, due to the challenging economic conditions, many doctors and nurses went to live and work abroad, but a lot of them are also employed in private healthcare organizations to earn higher salaries.

The *nurse scheduling problem* (NSP) is a combinatorial optimization problem to make rosters for nurses who provide 24-hour care service in a hospital by working in shifts. The rosters should satisfy various constraints such as required number of nurses at each shift, workable combination of nurses and requests for task assignments posed by individual nurses [2]. There are two cases regarding the NSP: the *static* and *dynamic*. The *static* NSP is to make a roster for a given scheduling horizon under given conditions and constraints. On the other hand, the *dynamic* NSP, is often called *nurse rerostering problem* (NRRP). However, unexpected events may occur in real life, which leads to schedule disruption and infeasibility.

The NRRP is to reconstruct or modify the predetermined roster for the current scheduling horizon aiming to respond to changing situations triggered by some kinds of emergencies. The most typical emergencies in hospitals are the following: (i) the need for greater number of nurses in one shift; (ii) the need for fewer numbers of nurses in one shift; (iii) absence of nurses for 1 day; (iv) sudden absence of a nurse for several consecutive days—the nurse gets sick. Since all these emergencies influence the fluctuation in the number of nurses, the schedulers should find a nurse who can fill up the vacancy of the absentee and modify some task assignments for the related nurses in the current roster.

This is usually an unpopular measure and it may even increase the personnel costs of the employer. Therefore, the criterion of NRRP typically involves the minimal number of changes of the original roster in these rescheduling processes because workers' private life is organized according to it and they must cancel or reschedule their already planned free-time activities. This staff allocation needs to balance service requirements with fairness and cost effectiveness [4].

This paper focuses on a new strategy based on hybrid approach to detecting the optimal solution in NRRP. The new proposed hybrid approach is obtained by combining fuzzy logic, ordered weighted averaging (OWA) empirical model and variable neighbourhood descent search. The model is tested with original real-world dataset obtained from the Oncology Institute of Vojvodina (OIoV), Serbia. This paper continues the authors' previous research on NRRP presented in [19] and their research in nurse decision-making, rostering and scheduling in healthcare organizations which are presented in [16], [17] and [18].

The rest of the paper is organized in the following way: Section 2 provides an overview of the basic idea and related work in NRRP. Section 3 presents hard and soft constraints, fuzzy logic, fuzzy OWA approach, variable neighbourhood descent search, the proposed algorithm for NRRP and representation of empirical dataset. Preliminary experimental results are presented in Section 4. Section 5 provides conclusions and some points for future work.

2. Basic idea in NRRP and related work

Due to their complexity and importance in real-world modern hospitals, NSPs have been extensively studied in both *operational research* and *artificial intelligence* societies for more than 40 years [2, 3]. The NSP is a well-known *nondeterministic polynomial acceptable problem* (NP), which stands for NP-hard scheduling problem that aims to allocate the required workload to the available staff nurses at healthcare organizations to meet the operational requirements and a range of preferences.

NSP consist of generating rosters where required shifts are assigned to nurses over a scheduling period satisfying many constraints [2, 3]. They are usually categorized into two categories: hard constraints and soft constraints, as defined below:

- *Hard constraints* must be satisfied to obtain feasible solutions for use in practice. A roster satisfying all hard constraints is usually termed feasible. A common hard constraint is to assign all shifts required to the limited number of nurses.
- *Soft constraints* are not obligatory but are desired to be satisfied as much as possible. A common soft constraint in NRPs is to generate rosters with a balanced workload so that human resources are used efficiently.

2.1. *Basic idea in NRRP*

One of the most obvious areas in a hospital environment is the automation of appointment, resource, scheduling and rerostering. It is not always recognized that nursing costs account for 50% of total hospital costs. Scheduling and rerostering have significant impacts not only on costs but also on nurses' job satisfaction [21].

In the past decades, many approaches have been proposed to solve NSP as they are manifested in different models. The NSP belongs to the domain of the human resources/personnel scheduling, but optimal solution derived from techniques with high computing times are usually less valuable than the ones based on a flexible algorithm or user intuitive application.

2.2. *Related work in NRRP*

Although NSP occurs in hospitals very often, the number of papers focused on NRRP is minor in comparison to the NSP. The first paper addressing NRRP was [12] from 2004. The authors proposed models based on the multi-commodity network flows that are expressed by an integer linear programming.

The study in [11] emphasized that the primary nurse preference to be observed in rerostering is to retain the original shift assignments as much as possible. An integer multi-commodity flow model was applied to solve the problem. In [13], a genetic algorithm approach was used to model and solve a case study problem, based on real data from a hospital setting. The authors generated problem sets with artificial complexity to test the algorithm efficiency and effectiveness. Evolutionary algorithm, population-based metaheuristics imitating the evolutionary ideas of natural selection and genetic processes, to solve the NRRP is presented in [8]. That paper considers multiple objectives with different priorities, including minimizing nurses' surplus and shortage, minimizing the penalties caused by nurses' preferences violations and deviations from the original roster and minimizing deviations from an average number of duties for each nurse. A *cooperative genetic algorithm* (CGA) is used to re-optimize the original schedule of the remainder of a current month after the occurrence of some changes at the beginning of the month. They defined a penalty function to minimize the changes between the original schedule and the re-optimized schedule for the remainder of the month [15]. Two parallel algorithms to solve the NRRP on a graphics processing unit are proposed in [1].

For effective nurse schedules, fuzzy theoretic evaluation approaches must be used to incorporate the fuzzy human preferences and choices. The present study seeks to develop a simulation of fuzzy multi-criteria evolution approach for the nurse rerostering problem as shown in [14]. Finally, more

FIGURE 1. The three phases of nurse scheduling.

related discussions on the NRRP, systematic literature review recognizing the existing rescheduling issues and mathematical modelling tools, are referred to in [4].

3. Modelling the NRRP

Modelling the NSP is the process of ensuring that there are always enough nurses, it comprises of numerous decisions based on different time horizons and different levels of details. These decisions can be divided into three planning phases, as illustrated in Figure 1.

The long-term planning is a part of the overall strategic planning process for each ward. First the ward managers must estimate how many nurses with each of the necessary skills are needed during all possible time periods of the day. When the staffing demand is known and when there is a given workforce of nurses, each nurse is assigned to a schedule specifying which shifts she should work, usually for a scheduling period of 4–10 weeks. This phase in the planning process can be referred to as the mid-term planning or nurse rostering. Whenever there is a shortage of nurses for a shift, the short-term planning consists of deciding whether to use overtime, to call in a nurse on her day off, to call in a substitute nurse or to try to manage despite the shortage.

This research is focused on NRRP problem on NSP in executive period, which presents dynamic NSP, in *intensive care unit* (ICU) at the OIoV. Duty rosters are now generated manually by head nurse for ICU, which enables the nurses to express their requests and preferences for working or not working certain shifts and days off. The proposed model introduces and combines two different metaheuristics techniques in decision-making in modelling NRRP: (i) *fuzzy OWA approach* and (ii) *variable neighbourhood descent search*.

3.1. Fuzzy OWA approach

The OWA approach is originally introduced to provide as a means for aggregating scores associated with the satisfaction to multiple decision-making criteria. Subsequently they have proved to be a useful family of aggregation operators for several different types of problems, and their application in generalizing decision making under uncertainty is particularly noted [20].

The OWA functions have the form $A(x) = \sum_{j=1}^{n} w_j \; x_{(j)}$, where $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)}$ and w_1, $w_2 ... w_n \geq 0$ with $\sum_{j=1}^{n} w_j$. These functions have been introduced by Yager in [20] and correspond to the Choquet integrals associated with symmetric capacities which are in detail presented in [5]. In the beginning, OWA operators which provide a general class of mean like aggregation operators are introduced, where OWA operator of dimension n is a mapping F: $R^n \rightarrow R$ which has an associated weighting vector is defined by Equations (1)–(3) [20]:

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ w_n \end{bmatrix} \tag{1}$$

$$w_j \in [0, 1], j = (1, n) \tag{2}$$

$$\sum_{j=1}^{n} w_j = 1. \tag{3}$$

Important classes of aggregation problems are generically referred to as multi-criteria aggregation problems. Examples of these problems arise in many applications such as decision-making, pattern recognition and information retrieval.

In fuzzy systems, the modelling of the aggregation can be obtained in the form of the importance transformation operator [7]. It can be assumed that F_{wa} is an OWA operator of dimension n with weighting vector W. Furthermore, it can be assumed that each of the arguments to be aggregated have an associated importance weight; thus, there is a collection of n pairs (u_j, a_j) where $u_j \in [0, 1]$ is the importance weight and $a_j \in [0, 1]$ indicates the score.

One approach to aggregation when we have importance weighted scores was to introduce some transformation G that converts the importance weights and scores into some effective value, $b_j = G$ (u_j, a_j), and then aggregate these effective scores using the aggregation operator. It appears natural to try to use this approach in the case of the OWA aggregation. Using this idea, the calculation should be

$$a^* = F_{wa}(b_1, b_2, \ldots, b_n). \tag{4}$$

The next step is to order the a_j's in descending order; it shall be denoted b_i as the ith largest of the a_j. Furthermore, let v_i denote the importance associated with the score that has the ith largest value. Let Q be a fuzzy subset on the unit interval corresponding to some linguistic quantifier. Now, the aggregated information can be considered as a collection of n pairs (v_i, b_i). The next step is to obtain the OWA weights associated with this aggregation:

$$w_i = Q\left(\frac{S_i}{T}\right) - Q\left(\frac{S_{i-1}}{T}\right) \quad \text{for } i = 1 \text{ to } n, \tag{5}$$

where $S_i = \sum_{k=1}^{i} v_k$ and $T = S_n = \sum_{k=1}^{n} v_k$.

Finally, it is possible to calculate the aggregated value a^* as

$$a^* = \sum_{i=1}^{n} b_i\, w_i. \tag{6}$$

3.2. Variable neighbourhood descent search

Variable neighbourhood search (VNS) is a metaheuristic proposed by some of the present authors a dozen years ago [10]. It is based on the idea of a systematic change of neighbourhood both in a descent phase to find a local optimum and in a perturbation phase to get out of the corresponding valley [6]. Originally designed for approximate solution of combinatorial optimization problems, it was extended to address mixed integer programs, nonlinear programs and recently mixed integer nonlinear programs.

VNS is based on three simple empirical facts: (i) a local minimum with respect to one neighbourhood structure is not necessarily so for another; (ii) a global minimum is a local minimum with respect to all possible neighbourhood structures; and (iii) local minima with respect to one or several N_k are relatively close to each other [9]. To solve (1) by using several neighbourhoods, facts 1 to 3 can be used in three different ways: (i) deterministic, (ii) stochastic and (iii) both deterministic and stochastic.

The solution move and neighbourhood change function is given in Algorithm 1. Function `NeighborhoodChange()` compares the incumbent value *f(x)* with the new value *f(x')* obtained from the kth neighbourhood (Line 1). If an improvement is obtained, the new incumbent is updated (Line 2) and k is returned to its initial value (Line 3). Otherwise, the next neighborhood is considered (Line 4).

Algorithm 1 Neighborhood change

Function `NeighborhoodChange` (x, x', k)
1 if $f(x') < f(x)$ **then**
2 $x \leftarrow x'$ //Make a move
3 $k \leftarrow 1$ //Initial neighborhood
else
4 $k \leftarrow k + 1$//Next neighborhood
end else
return x, k

The **variable neighborhood descent search** (VNDS) method is obtained if a change of neighborhoods is performed in a deterministic way. It is presented in Algorithm 2, where neighborhoods are denoted as Nk, $k = 1,\ldots,k$max. Most local search heuristics use a single or sometimes two neighborhoods for improving the current solution ($k_{max} \leq 2$) Figure 2.

Note that the final solution should be a local minimum, all k_{max} neighborhoods, and thus a global optimum is more likely to be reached than with a single structure.

Algorithm 2 Variable neighborhood descent

Function `VND` (x, k_{max})
$k \leftarrow 1$
repeat
$x' \leftarrow argmin_{y \in Nk(x)} f(y)$ //Find the best neighbor in $N_{k(x)}$
$x, k \leftarrow$ `NeighborhoodChange` (x, x', k) //Change neighborhood
until $k = kmax$
return x

3.3. *Hard and soft constraints*

Nurses in the unit have different skills categories, meaning different qualifications, specialization training, experience and gender, presented in Table 1. *Hard requests* define a constraint that must be respected in the roster and *soft constraints* define the preferred, desirable option expressed by a nurse. Some typical values for a few of the constraints are given below: (i) *Min* (*max*) nurses on shifts: 3 nurses in *day shifts*, 3 nurses in *night shifts*; (ii) it is not desirable to work a *night shift* followed by a *day shift*; (iii) after 5 *morning shifts*, 2 *days off* must be assigned; (iv) after a break of more than 7 days, *day shift* must be assigned; (v) at least one of the members of *shift* must be a *shift*

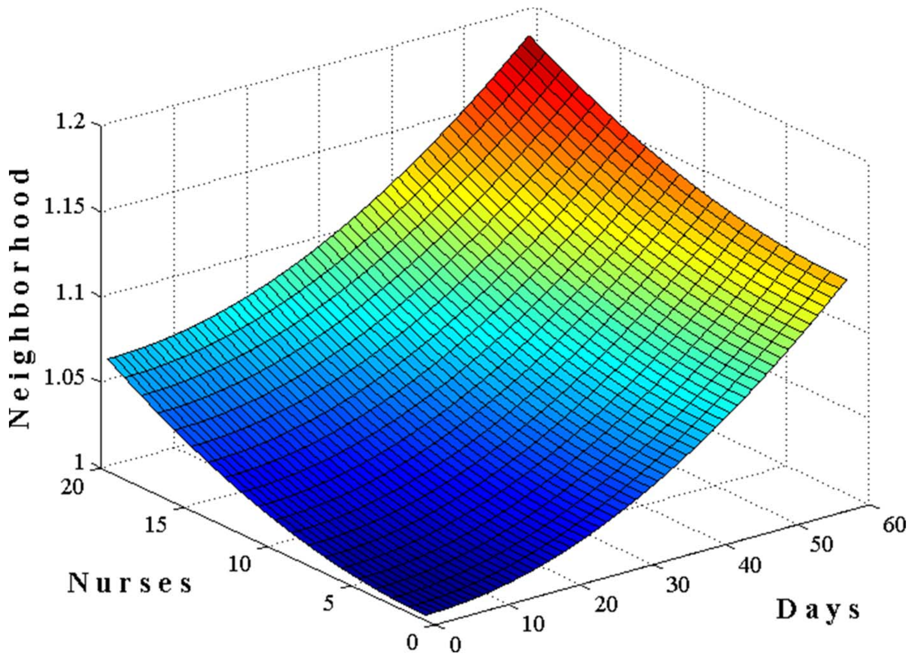

FIGURE 2. The variable neighborhoods descent search.

TABLE 1. Nurses' skill categories.

Nurse-ID	Years of service–experience	Shift leader	Specialization training	Nurse-ID	Years of service–experience	Shift leader	Specialization training
N-01	11	Yes	Yes	N-11	16	Yes	No
N-02	6	No	No	N-12	11	Yes	No
N-03	10	Yes	Yes	N-13	12	No	No
N-04	31	No	No	N-14	4	No	Yes
N-05	8	Yes	Yes	N-15	20	Yes	Yes
N-06	15	Yes	Yes	N-16	17	Yes	No
N-07	13	Yes	Yes	N-17	1	No	No
N-08	13	No	No	N-18	1	No	Yes
N-09	11	Yes	Yes	N-19	1	No	No
N-10	10	Yes	No	N-20	1	No	No

leader, according to their qualifications; (vi) *max* (*min*) days: full time nurses may not work more than pre-determined number of days.

The ideal and proposed work shift dynamic, which proposed management of OIoV, is as follows: *day–night–off–off–off* (DNOOO). Also, the DONOO dynamic is allowed, where there are two work shifts and three days off in 5 days are allowed as well. But, in the real word, more difficult shift dynamic is allowed: three working days and two days off (DNONO).

TABLE 2. Nurse-case duties, original dataset from 1 to 31 January and calculated nurse schedule from 1 to 28 February, calculation algorithm is presented in [19].

	January	**February**
	1 1 2 2 3 3	**1 2 2 2**
	1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1	1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8
1	DOONONOOODDOOODNOOODDNOODNOOOON	OOODNOOODNOOODNOOODNOOOYYYYY
2	DOODNOODBBCCBBBBBCCBBBBBBBBBBBB	VVVVYYYYYYYYYYVVVDNOOODNOOOD
3	ONOODONOONOODONNOODONOODNOOODON	OOODNOOODNOOODNOOODNOOODNOOO
4	OOBCCBBBBBCCYYYYYVVYYYYYVVYYYYY	VVVVYYYYYYYYYYYVVVVYYYYVDOODOD
5	ONOOODOODNNONONOOONOOODOODODODO	VVVVYYYYYYYYYYVVVDNOOODNOOOD
6	OOYVVYVYYYVVYVYYYVVXXNOOODNOOOO	DNOOODNOOODNOODNOOODNOOODNOO
7	NOODODOXXXONONOXNOOODXOOOODOODD	NOOODNOOODNOODNONOOODNOOODNO
8	ODNOOODONODDNOODNOOOODNOOOONOOD	NOOODNOOODNOOODNOOODNOOODNOO
9	DODDOOONODODOOODDNOOODNOOODNOOO	NOOODNOOODNOOODNOOODNOOODNOO
10	NOOODOODNONOODDOOODNOOODNOOODNO	OODNOOODNOOODNOOODNOOODNOOOO
11	VVYVVVYVYYYVVDNODODNOOODNOOODNO	OODNOOODNOOODNOODNOOODNOOODNO
12	VVYVVVYVYYYVVDDOOODOOODONOOODNO	OODNOOODNOOODNOOODNOOODNOOODN
13	VVYVVVYVYYYVVYYYYYVVNNOODOODOON	OOODNOOODNOOODNOODNOOODNOOODN
14	VVYVVVYVYYYVVYYYYYVVNNOODOODOON	OOODNOOODNOOODNOOOODNOOODNOOO
15	OODODXDXXXOOODNOODNOOODNONOONOO	DNOOODNOOODNOOODNOOODNOOODNO
16	OODOONOOVYVVYYYYYVVYYYYYVVNOODO	ODNOOODNOOODNOOODNOOODNOOODN
17	OOONNODNNOOODOOODNOOODNODOONOOO	DNOOODNOOODNOOODVYYYYVVYYYYY
18	OODOODNOODNNOOOODNOODNOOOONOOOD	VVVVYYYYYYYYYYVVVYYYYVVDNOOO
19	VVYVVVYVYYYVVYYYYYVVNOODOODOODOO	VVVVYYYYYYYYYYYVVVYYYYVVYYYYY
20	ODNOODOODODDOODNOOODNOOODNOOOON	OOODNOOODNOOOYVVVYYYYVVYYYYY

3.4. *Empirical dataset—graphical representation*

For this experiment, the original real-world dataset obtained between 1 January and 28 February 2014 for ICU of OIoV is used. Regular work days are 5 days per week, from Monday to Friday. Regular working hours are 7 hours and 12 minutes.

Nurses can work in three *on-duty* shifts: *day* (D) (06:30–18:30), *night* (N) (18:30–06:30) and *morning* (X) (07:30–14:42) shifts. There are also *free* shifts which include *day-off* (O), *sick days* (B), *maternity leave* (K), *annual leave on weekends* (V) and *annual leave* (Y); the abbreviations for all of these are used in Table **2**. Nurse-case duties, original dataset from 1 to 31 January and calculated nurse schedule from 1 to 28 February, is presented in Table **2**, and calculation algorithm is presented in [19].

3.5. *The proposed algorithm for NRRP*

The proposed algorithm for NRRP at the ICU at the OIoV is summarized in the following five steps. The basic steps of the proposed hybrid algorithm for NRRP are summarized by the pseudo code shown in Algorithm 3. Our algorithm is inspirited by integration of fuzzy logic method, OWA empirical approach and neighbourhood descent search. The OWA averaging approach is defined in Equation (1) all the way to Equation (6). The numerical values for fuzzy OWA are presented in Equations (7) and (8). The *workload nurse* is shown in Equation (9) [19].

$$\mathrm{W}^{\mathrm{T}} = [0.0785, 0.1285, 0.1787, 0.2286, 0.1787, 0.1285, 0.0785] \tag{7}$$

$$[\mathrm{D}, \mathrm{N}, \mathrm{X}, \mathrm{O}] \Rightarrow \mathrm{Q} = [1, 1, 0.6, 0] \tag{8}$$

$$workload = (day\_shift + night\_shift)\ /\ (day\_shift + night\_shift + day\_off) \tag{9}$$

$$workload\_nurse = \left(\sum_{i=1}^{n} W_i * Q_i * day\_penalty_i\right) + workload * workload\_penalty \tag{10}$$

$$workload\_nurse\_year = workload\_nurse + year * work\_penalty \tag{11}$$

$$workload\_nurse\_neigh = workload\_nurse\_year + neigh\_search * work\_penalty \tag{12}$$

The contributions of our presented research are the following: first, *workload_nurse_year* which takes care that older nurses work less in emergency situations than younger nurses which is presented in Equation (10); and second *workload_nurse_neigh* which takes care that distance between working days is as large as possible. For *workload_nurse_year, years of service–experience* has been introduced, which is presented in *Nurses' skill categories* (Table 1). For the *workload_nurse_neigh*, *neighbourhood descent search* approach has been introduced taking care that the distance between working days of nurses is as large as possible and it is shown in Equation (12).

$$\arg\ \max_{x}\ (\min)\ V(t, x)\ \ := x \in \{\ N, (D)\} \tag{13}$$

Partially target function, defined by *arguments of the maxima* (*arg max*), which occur in Equations (1)–(3), is presented in Equation (13), where *arg max* refers to the *inputs*, or arguments, at which the function *outputs* are as large as possible. The arguments of the maxima are the points of the domain of some function at which the function values are maximized where it is search for *night shifts*, but the *arguments of the minima* (*arg min*), refers to the *inputs* at which the function *outputs* are as smallest as possible when *day shifts* is searched.

4. Experimental results and discussion

Table 3 presents calculation for the nurse candidates for one new nurse in *day shift* for the 21 February based on the proposed hybrid algorithm. Three days before required rerostering day are used, as well as three days after the rerostering day. Table 3 presents available nurses, omitting the nurses N-01, N-03 and N-14, created *allowed list nurses* because it is impossible for a nurse to work *day shift* after *night shift*, which is one of the OIoV *hard constraints*. In this example *variable neighbourhood descent search* approach shows it's powerful, with attention to nurse *working day*. Nurse N-05 is selected due to *minimum daily workload* including *years of experience*, *variable neighbourhood descent search* approach with attention to nurse *working day* because *day shift* is required, which is presented in *final ranking*-**Position** column. Therefore, after applying the proposed algorithm for NRRP, *Position* 1 for N-05 is to be selected.

The *rank* changes of the three most important nurses for 21 February are presented in Table 4. It is interesting to notice that the nurses' rank changes when the number of elements increases. Rank (1) is given only with the nurse ID and types of nurse shifts (*day*, *night*, and *morning shifts*). Where *workload_2* is calculated as *workload* Equation (9) $*$ Column 8 from Table 3. Nurse N-05 in *rank (1)* is in position 3, but afterwards in *workload_2*, in *rank (2)* she is in position 2, and when *fuzzy OWA* is

Algorithm 3 The algorithm for NRRP

Begin

Step 1: — ***Initialization.***
Constraints hard/soft; **Nurse_number** = 20; **Nurse skills categories**;
Nurse-day duties = from 1 January to last day on duties;
Day = ReRostering day; **Require_shift** = (D/N); **W** = weighted matrix;
[D, N, X, O] = fuzzy weighted matrix ⇒ **Q** = [1, 1, 0.6, 0];
day_penalty = [80 90 100 0 100 90 80] for days;
workload_penalty = 100 penalty for workload; **work_penalty** = 100

Step 2: — ***Loop over all Nurses.***
for *j = 1:* **Nurse_number** *(all Nurses)* ***do***
find all nurses which shift is "O" for Day→List
remove from the List the nurses which have not allowed shift
define Allowed List
end for *j*
—***End Loop for Nurses.***

Step 3: — ***Loop over the Allowed List nurses.***
for *i = 1:* **Allowed_ nurses** *(Allowed List nurses)* ***do***
—Calculation weighted value
Define weighted matrix **W**
Equations (1), (2) and (3)
Define fuzzy weighted matrix **Q**
Equations (4), (5) and (6)
—End Calculation weighted value
Calculate **Workload_nurse** for **Allowed_ nurses usage OWA**
Equations (9), (10) and *(11)*
Calculate **Workload_nurse** for **Allowed_ nurses usage VNDS**
Equation (12)
end for *i*
—***End Loop for Allowed Nurses.***

Step 4: — ***Start for making decision.***
Sort**—Workload_nurse** for **Allowed_ nurses**
Decision**—Min/Max Workload (Nurse)**—depend of **Require_shift**
—***End of Decision.***

Step 5: **Post-processing the results and visualization.**

End.

included, its *rank (3)* is in position 2, and also after the nurse experience is included the nurse N-05 keeps the same position in *rank (4)*. Finally, after including VNDS in calculation the *final rank* for the nurse is position 1. This example shows, very graphically, how ranks for a selected nurse change when number of elements used for decision-making is increased.

Table 5 presents the whole calculation based on proposed hybrid model for nurse-case duties for a new nurse for *night shift* for 15 February. It lists all the available nurses omitting nurses N-11, N-15 and N-17 because it is impossible for a nurse to work *night shift* before *day shift* in the same day. In this example *neighbourhood descent search* approach shows it's powerful, with attention to nurse

TABLE 3. Nurse-case duties for a new nurse for *day shift*, after rerostering for 21 February, with the final solution and position.

Nu. ID	Pr. shift	Next shift	Day shift	Night shift	Morn. shift	Day-off	*((4) + (6))/(5)*	Work-load	Fuzzy OWA	Work. year	Work. neigh-bour	Position
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*	*(12)*	*(13)*
N-02	**O**	**O**	4	2	0	5	20.00	0.545	144.78	162.78	244.39	6
N-05	**N**	**O**	7	7	0	20	10.00	0.411	76.86	100.86	151.10	**1**
N-10	**N**	**O**	11	10	0	30	10.10	0.411	80.98	110.98	164.69	**3**
N-12	**O**	**D**	9	6	0	24	10.50	0.384	93.40	126.40	183.44	5
N-13	**O**	**O**	6	7	0	19	00.85	0.406	70.53	106.53	154.10	**2**
N-17	**O**	**D**	6	6	0	21	1	0.363	72.07	123.07	176.00	4

Nu. ID represents nurse identification number; **pr. shift**, previous working shift; **next shift**, next working shift; **day shift**, number of day shift; **morn. shift**, number of morning shift; **day-off**, number of regular day-off; **(4) + (6))/(5)**, ratio daily (day + morning) and night shifts; **workload**, nurse workload (Equation (9)); **fuzzy OWA**, fuzzy nurse workload which includes Fuzzy OWA (Equation (10)); **work. year**, fuzzy OWA nurse workload including years of experience (Equation (11)); **work. neighbour**, Fuzzy OWA nurse workload which include years of experience and VNDS approach, (Equation (12)); **position**, *final rank*.

TABLE 4. Nurse-case duties for a new nurse for *day shift*, after rerostering for 21 February, with the transitional results and their positions *final rank*.

Nu. ID	Work load	Rank (1)	Work load_2	Rank (2)	Fuzzy OWA	Rank (3)	Work. year	Rank (4)	Work. neighbour	Final rank
N-05	0.411	**3**	0.411	**2**	76.86	**2**	100.86	**2**	151.10	**1**
N-10	0.411	**3**	0.452	**3**	80.98	**3**	110.98	**3**	164.69	**3**
N-13	0.406	**1**	0.348	**1**	70.53	**1**	106.53	**1**	154.10	**2**

TABLE 5. Nurse-case duties for a new nurse for *night shift*, after rerostering for 15 February, with final solution and position.

Nu. ID	Pr. shift	Next shift	Day shift	Night shift	Morn. shift	Day-off	*((4) + (6)/(5)*	Work-load	Fuzzy OWA	Work. year	Work. neigh-bour	Position
	(2)	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*	*(12)*	*(13)*
N-10	**N**	**O**	10	9	0	26	1.11	0.422	82.62	52.62	71.58	**3**
N-12	**O**	**O**	8	5	0	20	1.60	0.393	98.72	65.72	87.19	**2**
N-13	**N**	**O**	5	6	0	15	0.83	0.423	73.87	46.53	46.59	4
N-14	**N**	**O**	10	10	0	25	1.00	0.444	83.73	61.87	104.81	**1**
N-16	**O**	**O**	5	5	0	17	1.00	0.370	72.72	21.72	28.39	5

working day. The selected nurse has four (4) *day-off* (O) after last *working day* and for *final ranking*, according to Equations (9), (10), (11) and (12) nurse N-14 is selected.

Section 2 presents the related work and most of the researches separately use deterministic, stochastic or metaheuristic methods and techniques such as integer linear programming, genetic algorithm, evolutionary algorithm and CGA. On the other hand, the scientific aspect, as well as the novelty of solutions in our research, is broad. First VNS is originally designed for approximate solution of combinatorial optimization problems, and it was extended to address mixed integer programs, nonlinear programs and recently mixed integer nonlinear programs. In generally, VNDS is based on three simple empirical facts: a local minimum, a global minimum and local minima with respect to one or several best neighborhood(s). Also, in experimental researches to solve real-world problems it can be used in three different ways: deterministic, stochastic or both deterministic and stochastic.

And second, the OWA approach is originally introduced to provide a means for aggregating scores associated with the satisfaction to multiple decision-making criteria. The most important classes of aggregation problems are generically referred to as multi-criteria aggregation and group decision-making problems. Examples of these problems arise in many real-world applications such as decision-making, pattern recognition and information retrieval in the most important forms: market, public and household of production goods or services.

5. Conclusion and future work

The aim of this paper is to propose the new hybrid strategy for detecting the best solution in NRRP. The new proposed hybrid approach is obtained by combining fuzzy logic, ordered weighted search and variable neighbourhood descent search. The model is tested with original real-world dataset obtained from the OIoV in Novi Sad, Serbia. The preliminary experimental results encourage authors' further research because the proposed hybrid system got logical, acceptable and applicable results. This model cares about human satisfaction—employed nurses—because scheduling and rerostering have significant impacts not only on employed costs but also on nurses' job satisfaction. The proposed model is not solely tied to the NRRP and it could be exploited in other real-world decision-making applications.

Our future research will focus on creating new model which will efficiently and cost-effectively solve NRRP. The new model will be tested with original real-world dataset for longer periods obtained from the Clinical Centre of Vojvodina in Novi Sad, Serbia. Also, it will be interested to test the proposed optimization algorithm in some production system.

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