A Particle Dyeing Approach for Track Continuity for the SMC-PHD Filter

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***Abstract*—This paper proposes a novel particle labeling (termed as 'dyeing') method for track continuity for the sequential Monte Carlo (SMC) implementation of the probability hypothesis density (PHD) filter. The Multi-Expected a Posterior (MEAP) estimator is employed to extract estimates that is of high accuracy and fast computing speed. In the estimate extracting process, particles are dyed by the color of the closest observation (different observations have different color) that corresponds to an estimate or clutter. The estimates of two successive scans are then associated with respect to their dyeing color interaction on the particles. Unlike the general labeling method, not all particles will be labeled to an estimate/track in the dyeing process. No modification is required to make on the PHD equation due to dyeing/MEAP. The proposed estimate association method is able to handle track initialization, termination, maintenance including track splitting and merging, based on observations of successive scans.**

***Keywords*—Multi-target tracking; sequential Monte Carlo; probability hypothesis density; track continuity; labeling**

I. Introduction

In recent years, growing success has been seen in applying finite set statistics (FISST) for Multi-target/object tracking (MTT/MOT) which deals with the detection and trajectory estimation of an unknown number of moving targets where available observations are noisy and are often mixture of the observations of targets (if they are detected) and the so-called clutter. FISST provides a concise yet adequate representation of the unordered targets and observations [1]. Based on FISST and the point process theory, the probability hypothesis density (PHD) filter [2] provides a tractable Bayesian solution, which has motivated different derivations, interpretations and implementations [3, 4, 5] recently. The PHD filter has been implemented in forms of weighted particles [6] or finite Gaussian mixtures (GM) [7, 8]. Specifically, the particle implementation that is often referred to as the SMC (Sequential Monte Carlo)-PHD filter is gaining special attention due to its ability to accommodate nonlinear dynamic and observation models and non-Gaussian noises, albeit with several disadvantages as compared to the GM implementation.

First, to maintain an adequate sample approximation, a large number of particles are usually required, which can often lead to heavy computational burden. This might be alleviated by using gating technology [9] or parallel computing [10] if available. Secondly, multi-estimate extraction (MEE) via unlabeled particles can be nontrivial. Typical solutions such as clustering are in fact unreliable and computationally intensive. We have recently demonstrated that this problem can be solved by using the Multi-Expected a Posterior (MEAP) estimator [11] that is free of clustering computation and is proven to be fast, accurate and reliable. Thirdly, the classical PHD filter only provides identity-free state-estimates of targets which are disassociated between successive scans. As a fundamental requirement of integrated tracking, it is often necessary to associate estimates for the same target over times and to identify the trajectory (namely 'track') of different targets. A track denotes a sequence of state-estimates over time, linked through the hypotheses that they belong to the same target. A couple of methods for estimate-to-track association, including track initialization, maintenance, termination, spawn, merging etc., have been reported. These methods, referred to as *track management* in this paper, can be classified into two groups.

The first group involves running an additional association method based on the estimates provided by the filter; e.g. using the PHD filter for pre-filtering the data input to an external filter such as a multiple hypothesis tracker(MHT). These are referred to as the "PHD-with-association" filter [12-15]. In this manner, the data association which seems to be graciously avoided in the filtering process has instead been moved outside the framework into the external algorithm [21]. This, to some extent, undermines the strength of the PHD filter. The other solution, as suggested by the terminology "track/particle labeling", augments the RFS of the state of target to be labelled [16-23], i.e., the state x of the target is augmented to be (l, x) where l is the track-specific identification label. This extension, although appearing to be systematically more concise, still inherently relies on a specific estimate-to-track association to determine the label l for each particle. The state of art track management methods are briefly reviewed in Section II.C of this paper.

This paper proposes a novel estimate-to-track association method for tracking continuity for the SMC-PHD filter, based on the MEAP estimator for multi-estimate extraction [11]. The proposed method, to a certain extent, is similar to the particle labeling method in the sense that particle labeling is realized through dyeing. But the difference is also significant. In general particle labeling implementations, each particle must have a label that is associated to one estimate/track. This is in fact unnecessary and restrictive since some particles may be wrongly weighted and maintained because of the presence of clutter but not the target and hence, they should not be labeled to any estimate. In our dyeing approach, the particle will not be associated to any estimate, if its likelihood is not strong enough and thereby is more likely existing because of clutter.

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The paper is arranged as follows. The technical background including basic contents of the SMC-PHD filter and the MEAP estimator and a brief review of the state-of-the-art research are provided in Section II. The proposed dyeing approach is described in Section III where its advantages over current methods are analyzed. Simulations are given in Section IV. Conclusion is given in Section V.

II. Background and Problem Statement

A. RFS representation and the PHD recursion

General multi-target trackers as well as the basic PHD filter require the following assumptions:

(A.1) Each target is assumed to evolve and generate observations independently of others;

(A.2) The clutter distribution is assumed to be Poisson and independent of the observations;

(A.3) One target can generate no more than one observation at each scan;

In addition, the PHD filter assumes that (A.4) the new appearing-target process is Poisson.

Let $\mathcal{F}(\mathcal{X})$ define the space of finite subsets of targets $\mathcal{X} \subseteq \mathbb{R}^{nx}$ and $\mathcal{F}(\mathcal{Z})$ define the space of finite subsets of observations $\mathcal{Z} \subseteq \mathbb{R}^{nz}$. Suppose that at time k, the collections of the states of targets are a RFS $X_k = \{x_{k,1}, \ldots, x_{k,N_k}\} \in F(\mathcal{X})$ where N_k is the number of targets, and the observations are a RFS $Z_k = \{z_{k,1}, \ldots, z_{k,M_k}\} \in F(\mathcal{Z})$ where M_k is the number of observations. The standard setup for MTT is given a multi-target RFS X_{k-1} at time $k-1$, each $x_{k-1} \in X_{k-1}$ either continues to exist at time k with survival probability $p_{S,k}(x_{k-1})$ and to move to a new state with a transition probability density $f_{k|k-1}(x_k|x_{k-1})$ or disappears with probability $1 - p_{S,k}(x_{k-1})$. At time k, a given target $x_k \in X_k$ is either detected with detection probability $p_{D,k}(x_k)$ and generates an observation $z_k \in Z_k$ with likelihood $g_k(z_k|x_k)$ or miss-detected with probability $1 - p_{D,k}(x_k)$.

Let $D_{k|k}$ and $D_{k|k-1}$ be the intensity (PHD) functions associated to the posterior and prior point processes: $f_{k|k}(x_k|Z_{1:k})$ and $f_{k|k-1}(x_k|Z_{1:k-1})$, namely $D_{k|k} = D_{k|k}(x_k|Z_{1:k})$, $D_{k|k-1} = D_{k|k-1}(x_k|Z_{1:k-1})$ where $Z_{1:k} = \{Z_1, Z_2 \ldots Z_k\}$. The PHD filter propagates the following Bayesian RFS recursions connected by predictors and updaters

$$\ldots \to D_{k-1|k-1} \to D_{k|k-1} \to D_{k|k} \to \ldots$$

The PHD predictor is

$$D_{k|k-1}(x) = \int_{\chi} \phi_{k|k-1}(x|u) D_{k-1|k-1}(u)\,du + \gamma_k(x) \tag{1}$$

where one abbreviation is used

$$\phi_{k|k-1}(x|u) = p_{S,k}(u) f_{k|k-1}(x|u) + b_k(x|u)$$

where $b_k(x|u)$ denotes the intensity function of the RFS $B_k(x|u)$ of targets spawned from the previous state u, and $\gamma_k(x)$ is the birth intensity function of new targets at scan k.

The PHD updater is

$$D_{k|k}(x) = \left(1 - p_{D,k}(x) + \sum_{z \in Z_k} \frac{p_{D,k}(x) g_k(z|x)}{\kappa_k(z) + C_k(z)}\right) D_{k|k-1}(x) \tag{2}$$

where $\kappa_k(z)$ denotes the clutter intensity at time $k$and

$$C_k(z) = \int p_{D,k}(u) g_k(z|x) D_{k|k-1}(u)\,du \tag{3}$$

B. A component view of the particle weight in SMC-PHD filter

The SMC-PHD filter employs a set of particles associated with weights to approximate the PHD predictor and updater. Given the importance densities $p_k(\cdot\,|Z_k)$, $q_k(\cdot\,|x_{k-1}, Z_k)$ and assuming there are L_{k-1} particles in time $k-1$ and J_k new particles are allocated for possible new-born targets, the particle approximation of the predictor $D_{k|k-1}$ can be written as

$$D_{k|k-1}(x_k) = \sum_{i=1}^{L_{k-1}+J_k} w_{k|k-1}^{(i)} \delta_{x_k^{(i)}}(x_k) \tag{4}$$

where the particle state and weight are given as

$$x_k^{(i)} \sim \begin{cases} q_k\left(\cdot \middle| x_{k-1}^{(i)}, Z_k\right), & i = 1, \ldots, L_{k-1} \\ p_k\left(\cdot \middle| Z_k\right), & i = L_{k-1}+1, \ldots, L_{k-1}+J_k \end{cases} \tag{5}$$

$$w_{k|k-1}^{(i)} = \begin{cases} \dfrac{\phi_{k|k-1}\left(x_k^{(i)} \middle| x_{k-1}^{(i)}\right) w_{k-1}^{(i)}}{q_k\left(x_k^{(i)} \middle| x_{k-1}^{(i)}, Z_k\right)}, & i = 1, \ldots, L_{k-1} \\ \dfrac{\gamma_k\left(x_k^{(i)}\right)}{J_k p_k\left(x_k^{(i)} \middle| Z_k\right)}, & i = L_{k-1}+1, \ldots, L_{k-1}+J_k \end{cases} \tag{6}$$

where $\delta_x(\cdot)$ denotes the delta-Dirac mass located in x.

The particle approximation of the PHD updater $D_{k|k}$ can be written as

$$D_{k|k}(x_k) = \sum_{i=1}^{L_{k-1}+J_k} w_{k|k}^{(i)} \delta_{x_k^{(i)}}(x_k) \tag{7}$$

where,

$$w_{k|k}^{(i)} = \left(1 - p_{D,k}\left(x_k^{(i)}\right) + \sum_{z \in Z_k} \frac{p_{D,k}\left(x_k^{(i)}\right) g_k\left(z \middle| x_k^{(i)}\right)}{\kappa_k(z) + C_k(z)}\right) w_{k|k-1}^{(i)} \tag{8}$$

$$C_k(z) = \sum_{j=1}^{L_{k-1}+J_k} p_{D,k}\left(x_k^{(j)}\right) g_k\left(z \middle| x_k^{(j)}\right) w_{k|k-1}^{(j)} \tag{9}$$

Correspondingly, we define its decomposition unit as

$$c_k(z, j) := p_{D,k}\left(x_k^{(j)}\right) g_k\left(z \middle| x_k^{(j)}\right) w_{k|k-1}^{(j)} \tag{10}$$

Also, the weight of particles can be taken apart as

$$w_k(z, j) := \begin{cases} \dfrac{p_{D,k}\left(x_k^{(j)}\right) g_k\left(z \middle| x_k^{(j)}\right) w_{k|k-1}^{(j)}}{\kappa_k(z) + C_k(z)}, & z \in Z_k \\ \left(1 - p_{D,k}\left(x_k^{(j)}\right)\right) w_{k|k-1}^{(j)}, & z = z_0 \end{cases} \tag{11}$$

where the symbol z_0 is introduced for consistency to represent the missed observation(s). $w_k(z, j)$ for $z \in \{z_0, Z_k\}$ are referred to as the weight components in the paper. From the definition, it is straightforward to obtain for each particle j,

$$w_{k|k}^{(j)} = \sum_{z \in \{z_0, Z_k\}} w_k(z, j) \tag{12}$$

The sum of the weight components of all particles for each $z \in \{z_0, Z_k\}$ can be defined as

$$W_k(z) := \sum_{j=1}^{L_{k-1}+J_k} w_k(z,j) \quad (13)$$

In a dual form, we have

$$\sum_{j=1}^{L_{k-1}+J_k} w_{k|k}^{(j)} = \sum_{z \in \{z_0, Z_k\}} W_k(z) \quad (14)$$

The PHD represents the local density of the distribution of targets and its integration in a region gives the expected number of targets in that region. In the SMC implementation, the expected number $\widehat{N}_k$ of targets can be determined based on the total weight mass $W_k = \sum_z W_k(z)$ as

$$\hat{N}_k = [W_k] = \left[\sum_{z \in \{z_0, Z_k\}} W_k(z) \right] \quad (15)$$

where $[\cdot]$ gives the nearest integer around the content.

C. MEAP Estimator

Estimate extraction is a prerequisite for track management but multi-estimate extraction (namely MEE) is a nontrivial task for the SMC-PHD filter. The commonly used clustering method suffers from problems of high computational complexity and the result is unreliable. In contrast, the multi-EAP (MEAP) estimator [11] extends the EAP estimator for MEE that employs a type of "divide and conquer" solution based on particle-to-observation association. According to (A.3) and regardless of extended targets, each estimate corresponds to one observation, and one observation has no more than one corresponding estimate. So, we have the following definition

Definition 1 An observation is called estimate-observation if it corresponds to one final estimate in MEAP and is called as spurious observation if it corresponds to no estimate.

The MEAP estimator distinguishes estimate-observations from clutter (spurious observation) first and then for each estimate-observation, one estimate can be extracted by employing the EAP (Expected a Posterior) estimator, as shown in Algorithm 1. In Algorithm 1, $\Xi(a)$ is the RFS of particles that are associated to observation a based on the near and nearest neighbor (NNN) principle. The details can be found in [11].

MEAP is approximately a Minimum Mean Square Error (MMSE) estimator, rendering high estimation accuracy. Secondly, it eliminates iterative clustering resulting in fast and reliable computation (which is also excellently suitable for parallel processing [10]). Our estimate-to-track association is processed within the MEAP estimator throughout.

Algorithm 1 MEAP estimator for the SMC-PHD filter

FOR $j = 1, \ldots, \min(\widehat{N}_k, |Z_k|)$ DO

$$a = \arg\max_z \{W_k(z)\}_{z=1}^{|Z_k|} \quad (16)$$

$$x_j^{\mathrm{EAP}} = \frac{\sum_{i \in \Xi(a)} g_k\left(a \middle| x_k^{(i)}\right) w_{k|k-1}^{(i)} x_{k|k-1}^{(i)}}{\sum_{i \in \Xi(a)} g_k\left(a \middle| x_k^{(i)}\right) w_{k|k-1}^{(i)}} \quad (17)$$

$$W_k(a) = 0$$

D. State-of-the-art track management for the PHD filter

Although multi-state estimate can be well extracted by the MEAP estimator in the PHD filter, additional estimate-to-track association are required for track continuity including track initialization, maintenance, termination, spawn/splitting and merging. There are primarily two classes of solutions to associate the detections/estimate to separate targets/tracks between successive scans.

The first group of solutions involves an independent complement to the filter. For example, when the PHD filter executes filtering as usual, its output of multi-estimate is used as the input data for the two-dimensional assignment [12] for a multiple hypothesis tracker[13], which can be of a high complexity in both time and space. Obviously, other basic association methods such as nearest neighbor and joint probabilistic data association (JPDA) can also be employed. For example, joint maximum likelihood probabilistic data association (JML-PDA) is used for initializing the tracks while JPDA is used for recursive track maintenance [14], which seamlessly share information between the initialization and maintenance stages of the tracker. In addition, the results of the estimation are used as vertexes to construct a connectivity graph with associated weights, and the cross entropy technique is employed as a global optimization scheme to calculate the optimal feasible associated events [15]. Here the PHD filter essentially acts as a de-cluttering tool for the external filter. The data association problem, which is circumvented in the PHD filter, is simply moved outside the framework into the external algorithm, albeit with reduced computational complexity.

Another group of track management methods can be referred to as the *labeling* or *tagging* method that is more formally modelled by augmenting the RFS of the state of targets to be labelled in [23] in which the PHD recursions are rewritten. Generally, the state x of the target is augmented to be (l, x) where l is the track-specific identification label. These labels are propagated and maintained together with the particles or Gaussian components over time to continually associate estimates. This fundamental extension of the PHD recursion is systematically concise that enjoys the "soft-resolving" property [24] and "Bayesian optimal" property [28]. An idea first proposed in [18] partitions the particle data in order to assign labels to the particles within the same cluster and associate the clusters between scans if there is a large intersection of particles with the same label from the previous scan. A similar method proposed in [19] is derived for the GM-PHD filter, where the Gaussian components are labeled. The labeling method may lose to the PHD-with association method [13] especially for nearby or cross track, although it is much computational cheap. In order to improve the estimate-to-track association quality, the state estimates of multiple scans may be taken into account rather than only two successive scans [15, 26] or the velocity information of the state might be taken into account [22]. Specifically, the hybrid model [22] uses the label when targets are far apart mutually, and uses the velocity information when targets are close to or cross each other, although the switching itself is only an ad-hoc case. Further evaluation is performed on Labels for improvement [29].

Most existing track management methods do not include track merging and splitting. Nonetheless, there is an exception. A number of ad-hoc thresholds are developed in [27] for distinguishing between track existing, initiation and termination as well as track merging and splitting. However, the limitations of the current two types of track management solutions can be summarized as follows:

1) The PHD-with-association method does not take advantage of associating estimates in the process of filtering, and it is a type of hard-resolving association that has difficulty in dealing with track cross, merging and splitting. This can be partly resolved by the labeling method.

2) In the conventional labeling method, all particles must be associated to an estimate and thus have a label. This is unnecessary and can be problematic as many particles exist because of the presence of clutter and should not be associated to any estimate (otherwise it will confuse the track association).

3) Most methods depend on clustering for MEE which is proven to be much inferior to the MEAP estimator in terms of computing speed and estimation accuracy. The MEE difficulty forms one of the main challenges for track management for the SMC-PHD filter as compared for the GM-PHD filter.

In the effort to address these restrictions and disadvantages, we propose a novel particle labeling approach called 'dyeing' based on the MEAP estimator in the following.

III. MEAP-BASED DYEING SYSTEM FOR PARTICLE LABELING

A. *Dyeing system for particle labeling*

Our proposed approach associates estimates between successive scans based on the dyeing-color interaction of the *estimate-observations* to the particles, where particle labeling is realized by dyeing particles by the color of estimates. In brief, the track management approach consists of two parts:

1) ***Dyeing***. Given that different observations have been associated with different colors (see Rule 1) and the estimate has the same color with its corresponding estimate-observation (see Rule 2), then, the color of estimate-observation is used to dye subsequent particles (see Rule 3). As a result of dyeing, all particles are dyed/labeled with a new color that is same as the corresponding estimates or, will have no color if it is contributed more by one spurious observation.

The dyeing method is performed via Rules 1-3 (to determine the color of observations, estimates and particles respectively). Fig. 1 illustrates one case of the dyeing process. The height of the cylinder represents the weight value, while the weight of the particles is composed of different components regarding different observations $z \in \{z_0, Z_k\}$, as shown in (12). Initially, there are four particles associated to one estimate (yellow) and one new-born particle with no color at time $k-1$. At time k, all observations are used to update particles in which z_1 and z_3 correspond to two estimate-observations with corresponding colors (red and purple) while spurious observation have no color. As a result, the corresponding particles are dyed with the color of z (including no color), where z satisfies

$$z = \max_{z \in Z_k} \left(w_k \left(z, j \right) \right) \tag{18}$$

2) ***Association***. After dyeing, estimate-to-track association is performed by matching the interaction of the original color and the new dyeing color on particles to associate the estimates between two successive scans, including track initiation (see Rule 4), maintenance (see Rule 5), spawn/splitting (see Rule 6), merging (see Rule 7) and termination (see Rule 8). Especially, two particles, neither one with color, are thought/treated as of the same color (referred to as no-color).

In the association rules proposed, we need to compare the association possibility. Since a resampling that equally weight all particles [30] will be executed in each iteration of the filter, the weight of particles is homogeneous before PHD updating. This enables us to count only the number of particles for comparison in Rules 4-8. Otherwise, the weight of the particles needs to be taken into account with the number of particles.

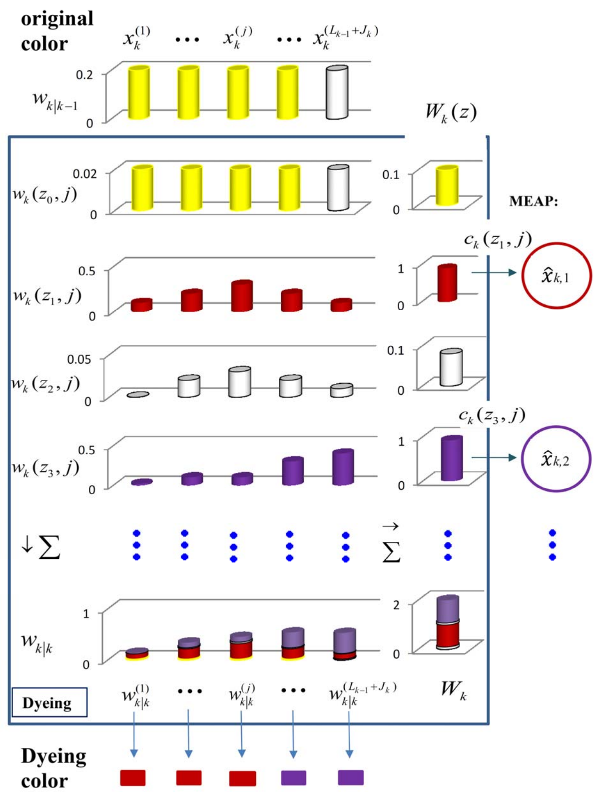

Fig.1 Particle dyeing in the MEE process based on MEAP

Rule 1 (*color of observation*). The color of different estimate -observations is specified to be different with one another at each scan and independently/unrelated between different scans. Spurious observations have no color.

Rule 2 (*color of estimate*). Each state-estimate $\hat{x}_{k,i}$ obtained by the MEAP method corresponds to one observation $z \in Z_k$ (namely estimate-observation) and it will be labeled the same color with the corresponding estimate-observation.

Rule 3 (*color of particle*). The color of particles are dyed in three sub-rules:

(R3.1) New-born particle has no color.

(R3.2) In resampling, each resampled particle will inherit the color of the original particle (including 'no color' case).

(R3.3) During the process of weight updating (as shown in fig.1), each particle will be dyed with the color of the observation (including 'no color') that contributes most to its weight. That is, particle j will be dyed the same color with observation $a = \max_{z \in Z_k} w_k(z, j)$ (the NN association).

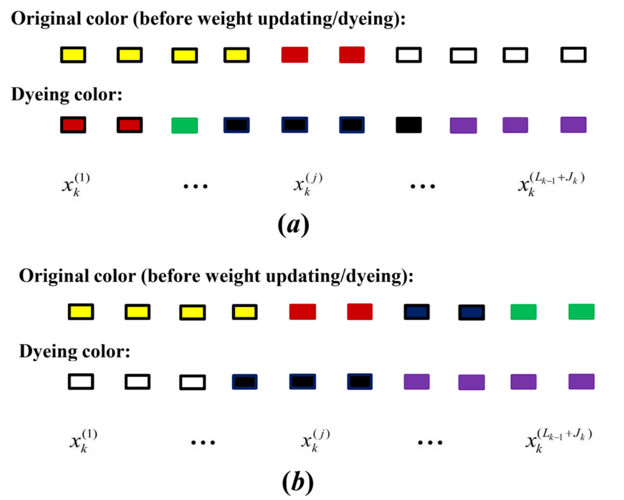

Fig.2 Two examples for estimate-to-track association via dyeing color

Rule 4 (*track initiation*). If the color of one estimate has been dyed on new born particles more than on existing particles (with color), that estimate will be initialized as a new track (e.g. the purple estimate is a new track in Fig.2 (*a*) if the upper particles of no color are mostly new-born). Information about target birth [31] might be integrated in this operation. The following is a sub-rule to identify possible false estimate.

(R4.1) If the color of one estimate has been dyed on particles (that have no color but are not new born particles) more than particles of color, that estimate is very likely to be a false estimate (e.g. the purple estimate is a new track in Fig.2 (*a*) if none of the upper particles of no color is new-born).

Rule 5 (*track maintenance*). If the color of one estimate $\hat{x}_{k,j}$ has been dyed mostly on particles that are of the same color as the estimate $\hat{x}_{k-1,i}$ which has not been associated to another estimate $\hat{x}_{k,l}, l \neq j$, then these two estimates $\hat{x}_{k,j}$ and $\hat{x}_{k-1,i}$ from successive frames will be associated with the same track (e.g. the black estimate will be associated with the red track in Fig.2 (*a*) and (*b*)).

Rule 6 (*track spawn/splitting*). Based on Rule 5, if several estimates $\hat{x}_{k,j}, j = 1, \ldots$ at time k, are associated to the same estimate $\hat{x}_{k-1,i}$ at time $k-1$, these estimates $\hat{x}_{k,j}, j = 1, \ldots$ will be associated to the same previous track $\hat{x}_{k-1,i}$ to generate several new tracks, i.e. track spawn/splitting (e.g., the green and red tracks are spawn from the yellow track in Fig. 2 (*a*)).

Rule 7 (*track merging*). Based on Rule 5, if several estimates $\hat{x}_{k-1,j}, j = 1, \ldots$ at the time$k-1$, are associated to the same estimate $\hat{x}_{k,i}$ at time k, the tracks $\hat{x}_{k-1,j}, j = 1, \ldots$ should be merged to be one track $\hat{x}_{k,i}$ i.e. track merging (e.g. the blue and green tracks will be merged to the purple one in Fig.2 (*b*)).

Rule 8(*track termination*). If most (e.g. more than a half) of the particles that are of the same color with one estimate at time *k*-1: $\hat{x}_{k-1,i}$ have been dyed with no color at the time k, the estimate $\hat{x}_{k-1,i}$ will terminate (e.g. the yellow track will be terminated in Fig. 2 (*b*)).

B. Remarks

In our approach, labeling is realized via dyeing with regard to individual observations, where the label for each particle is represented by a color. However, we do not have to label all particles as *some particles do not belong to any estimate but they actually exist because of the presence of clutter*. Because of this, the dyeing is less affected by the clutter and is expected to be more reliable especially in low signal-noise-ratio scenes. In addition, the MEAP estimator is more accurate than clustering, and the resulting track management is expected to contract high reliability and accuracy. On the other hand, instead of augmenting the label with the state of particles, we treat the estimate-to-track as an independent but closely connected process to the PHD equation, which means that there is no need to make any change to the PHD equations. All of these render reliable performance and fast computation. Especially, the estimate-to-track association method proposed is capable to distinguish track spawn/merging and splitting, given an adequate number of particles being used. However, there are still several limitations with the raw version of our approach due to the dilemma of the associated problems.

Remark 1 It is still of great challenge to distinguish track merging and termination as long as the only information used is limited. For example in the scenario of Fig. 3, it is both possible that tracks a and b are merged at point C and one of them is terminated. Furthermore, the standard PHD does not include such a term for target merging. In order to distinguish this, more information such as the target's physical shape (extended target modelling [25]) or moving velocity might be helpful [16]. In the current stage, we will not consider extended target and have not included the consideration of velocity of targets yet. To combat with this situation, the additional information will be necessary and helpful.

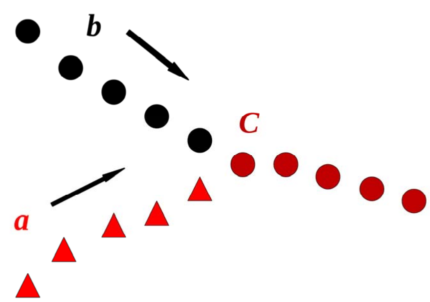

Fig. 3 A confusing case: track merging or termination?

Remark 2 It is also of challenge to deal with miss-detection of targets and false estimation. When a target is miss-detected, the estimated trajectory of that target will often be broken into two parts (see Fig. 4; there is a miss-detection at point A) and when there is a false estimate, one false track will often be created. To solve these, existing solutions based on multiple frame information rather than only two successive scans are applicable. For example, observations of more than three scans are required to confirm a real track [15, 26] and to compensate miss-detection by smoothing (to associate two tracks that is broken by a miss-detection) [16, 27, 24, 29]. However, it is necessary to note that it is possible that there are just in fact two different targets in Fig. 4. This scenario just exposes the weakness of the PHD filter [3, 28]. As the future work, advanced dyeing approaches will integrate solutions that are efficient at coping with miss-detection and that employ observation information of multiple scans.

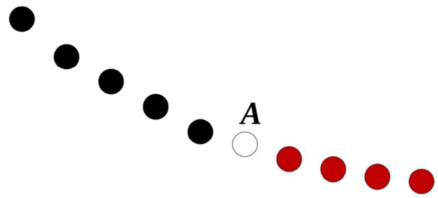

Fig. 4 A confusing case: two separate tracks or one broken track?

IV. Simulations

A. Scenario 1

The first simulation is designed in a two-dimensional scenario over the region [-100,100]×[-100,100] (the unit is omitted here). First, a horizontally moving target crosses a vertically moving target where the state comprise planar position and velocity $x_k = [p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}]$. Secondly, four moving targets cross each other. These targets appear according to a Poisson point process with intensity function $\gamma_i = 0.1\mathcal{N}(.; B_i, Q)$, where B_1= [-72, 3, 0, 0]^T, B_2=[0, 0, 75, -3]^T, B_3= [-62, 2.6, 36, -1.5]^T, B_4= [-36, 1.5, 62, -2.6]^T, Q=$diag$([5, 2, 5, 2]^T), $diag(a)$ gives a diagonal matrix with diagonal a. The simulated target dynamics is given as

$$x_k = \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix} x_{k-1} + \begin{bmatrix} \Delta^2/2 & 0 \\ \Delta & 0 \\ 0 & \Delta^2/2 \\ 0 & \Delta \end{bmatrix} \begin{bmatrix} w_{1,k} \\ w_{2,k} \end{bmatrix} \quad (19)$$

where the sampling time Δ=1, the processing Gaussian noise $w_{1,k} \sim \mathcal{N}(0,1)$, $w_{2,k} \sim \mathcal{N}(0,0.1)$. Furthermore, the position-only observation equation is given by

$$z_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} v_{1,k} \\ v_{2,k} \end{bmatrix} \quad (20)$$

with the Gaussian noise $v_{1,k} \sim \mathcal{N}(0,2.5)$, $v_{2,k} \sim \mathcal{N}(0,2.5)$.

Each existing target has a target survival probability $p_S(x)$=0.95 and a detection probability $p_D(x)$=0.95. Clutter is uniformly distributed over the region with an average rate of r points per scan, i.e. $\kappa_k = r/200^2$. Furthermore, 500 particles per expected target are used and the total number of particles is hard-limited to be not less than 300. To provide the insight of the method, the dyeing estimate-to-track method applied is a straightforward version that reserves isolated track and broken track in the outcome, and without any extra assistance. We iterate that track smoothing is necessary in practice.

Based on the MEAP estimator, the OSPA miss-distance (cut-off c=100, order p=2 [32]) and estimated number of targets are plotted in Fig. 5 for r=10. By using the proposed dyeing association method, the estimated tracks in the scenario are plotted in Fig. 6 where different color indicates different tracks. The results show that our method is able to initialize, maintain and terminate tracks however, miss-detection of moving targets has broken the track (see Remark 2) and false tracks can be generated (some of the isolated points marked in red) due to false state-estimation of targets. In a more complicated track-cross scenario of four moving targets, the estimated number and trajectories of targets are shown in Fig. 7 and 8. The results confirm that the proposed dyeing method work for close and cross continuous track and the MEAP estimator enjoys higher estimation accuracy than k-mean clustering. However, as aforementioned the observation of only two successive scans is inadequate to deal with false alarm and miss-detection for estimate-to-track association. Further technologies proposed in [15, 16, 24-27, 29] can be applied within our approach.

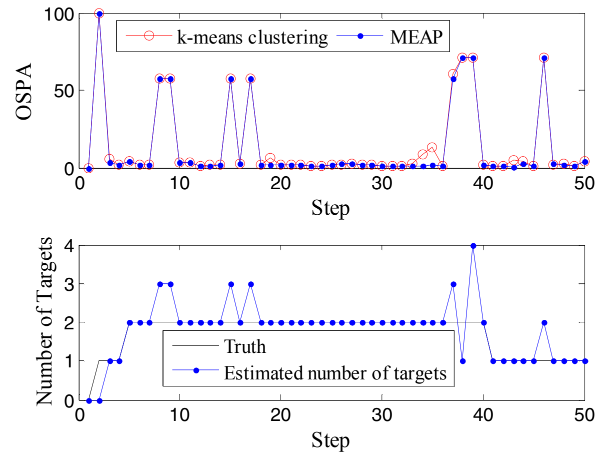

Fig. 5 OSPA and estimated number of targets when r=10

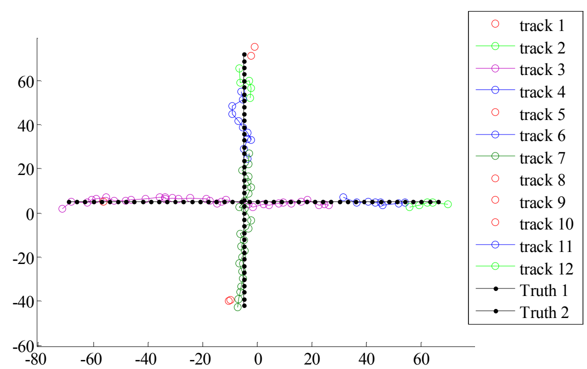

Fig. 6 Track continuity when r=10

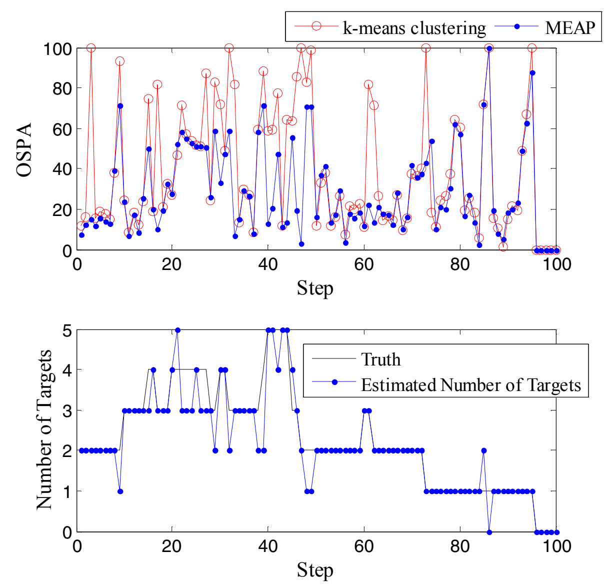

Fig. 7 OSPA and estimated number of targets when r=10

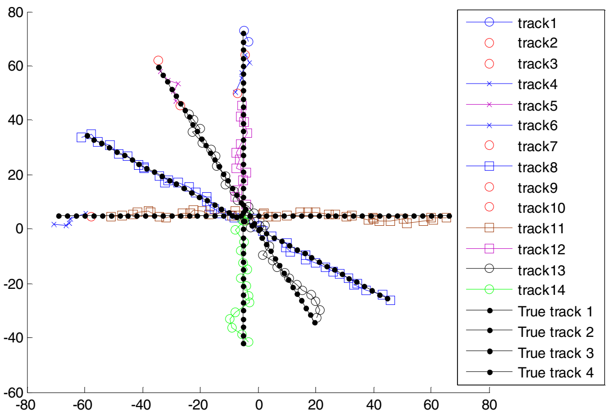

Fig. 8 Track continuity when r=10

B. Scenario 2

In this simulation, new targets spontaneously appear from four different areas and the observation consists of range and bearing information of targets. The trajectories of targets simulated are plotted in Fig.9 where the color distinguish different birth model and each trajectory starts from '∆' and ends at '□'. In total, ten targets randomly appear from these four target birth places and move in the region. The target birth process follows a Poisson RFS with intensity $\gamma_k = \sum_{i=1}^{4} r_{k,i} N(\cdot; B_i, Q)$, where $B_1 = [-1500,0,250,0]$, $B_2 = [-250,0,1000,0]$, $B_3 = [250,0,750,0]$, $B_4 = [1000,0,1500,0]$, Q =diag $([50,50,50,50]^T)^2$ and $r_{k,1} = 0.02$, $r_{k,2} = 0.02$, $r_{k,3} = 0.03$, $r_{k,4} = 0.03$. Each target moves according to the Markov transition as given in (19) but differently, the process noise $w_{1,k} \sim \mathcal{N}(0,15)$, $w_{2,k} \sim \mathcal{N}(0,15)$.

The range-and-bearing observation region is the half disc of radius 2000m. The observation is a noisy range and bearing vector given by

$$z_k = \begin{bmatrix} \sqrt{p_{x,k}^2 + p_{y,k}^2} \\ \arctan(p_{x,k} / p_{y,k}) \end{bmatrix} + v_k \tag{21}$$

where $v_k \sim \mathcal{N}(\cdot; 0, R_k)$, with $R_k = \mathrm{diag}([\sigma_r^2, \sigma_\theta^2]^T)$, $\sigma_r = 5$m, $\sigma_\theta = \pi/180$rad/s. Clutter is Poisson with intensity κ=1.6×10^{-3} that is an average rate of 10 points per scan over the region [0, 2000]m ×[0, π]rad.

The simulation uses a target survival probability $p_S = 0.99$, and a target detection probability function $p_{D,k}(x) = 0.98 \times \mathcal{N}([p_{x,k}, p_{y,k}]^T; 0, 6000^2 I_2)/\mathcal{N}(0; 0, 6000^2 I_2$. 1000 particles per expected target are used and the minimum number of particles is hard-limited to be not less than 600.

The OSPA miss-distance, estimated number of targets and tracks are given in Fig. 10 and 11 respectively. The results show that MEAP obtains obvious advantage over k-means on estimation accuracy. There are 34 tracks generated in total for the real 10 target trajectories, in which as many as 14 tracks are merely isolated estimates that are mainly generated due to false estimation. There are also broken tracks due to miss-detection as shown by different color on the same target trajectory. Further technologies are therefore necessarily required to filter out isolated estimates from the track result and to incorporate two successive tracks that are highly possible broken from one track (these can be done via some ad-hoc strategies), then the final track result will be much better. The results have demonstrated that the proposed estimate-to-track association method works efficiently for track continuity, which can serve as a preferable substitute to the general labelling approach because of the less influence by clutter and the high reliability and estimation accuracy of the MEAP estimator.

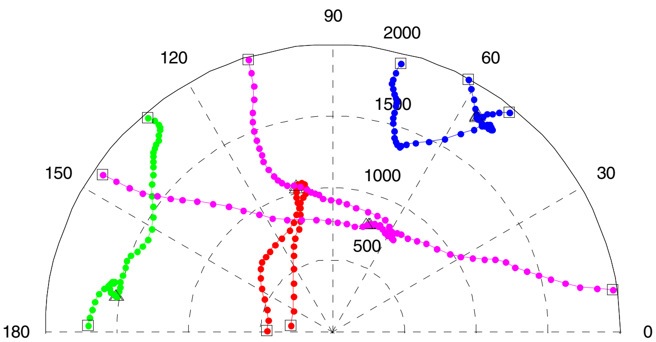

Fig. 9 True trajectories of targets generated from four birth models

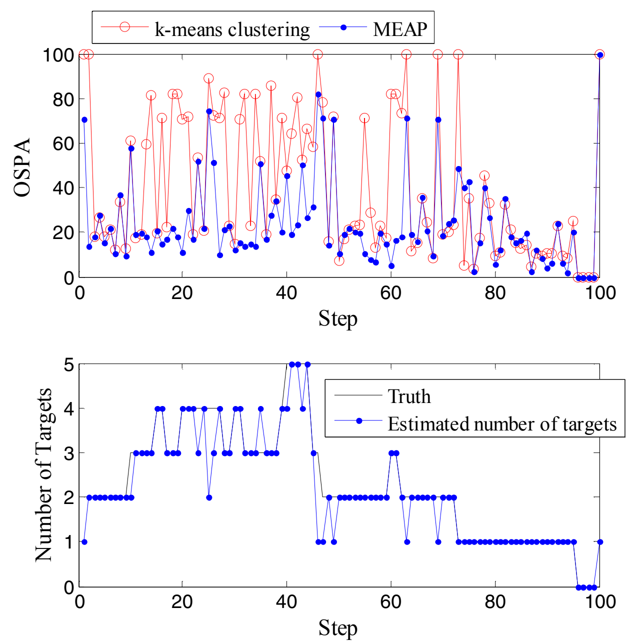

Fig. 10 OSPA and estimated number of targets when r=10

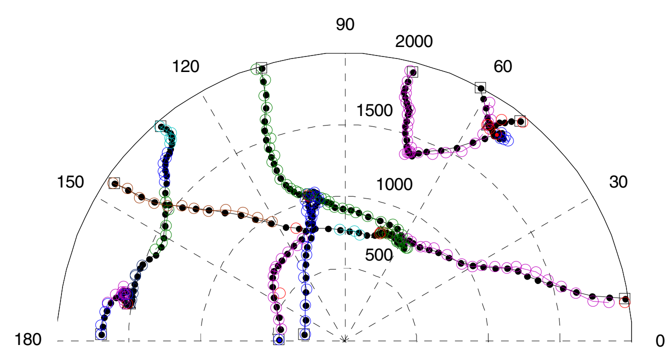

Fig. 11 Track continuity when r=10

V. CONCLUSION

The Multi-Expected a Posterior (MEAP) estimator provides high reliability and estimation accuracy and fast computation for multiple estimate extraction in the SMC-PHD filter. Based on MEAP, the dyeing approach has been proposed to handle track initialization, termination, maintenance including track splitting and merging for track continuity in the SMC-PHD filter. Our approach takes advantage of the soft-association property provided by the particle cloud. Unlike the general labeling method, not all particles will be labeled in the dyeing process and the label/color of particles are straightforward to determine. In the dyeing process, particles might be dyed with the color that is the same with one estimate or have no color. The estimates obtained by the MEAP estimator between two successive scans can now be associated based on the dyeing color interaction of the particles. However, there are still scenes which are hard to distinguish between track splitting and track birth, between track merging and termination based on observations of two successive scans only.

The future work will be emphasizing on track smoothing technologies by combining the dyeing approach with further information of multi-frame observations and/or target moving velocity (or other extended property). On the other side, the result of track is useful to enhance/correct the estimates of the tracker, e.g. the number of continuous tracks can provide careful information about the number of targets, which can be feed backed to the tracker.

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